MATH 211 SPRING 2013 **TEST II VERSION A**

NAME:

SECTION:

(1) True/False Questions: (1 point each).

T F $f(x)$ is $O(1)$ means that $f(x)$ is a constant function.

T F If $f(x)$ is $\Omega(g(x))$ then there exist unique constants C and k such that $|g(x)| < C|f(x)|$ for all $x > k$.

T F For any positive integers A, b and m, $(a \mod m)+(b \mod m) = (a+b \mod m)$

T F It is not possible to find functions $f(x)$ and $g(x)$ such that $f(x)$ is $O(g(x))$ and $g(x)$ is $O(f(x))$.

T F If $f_1(x)$ is $O(g_1(x))$ and $f_2(x)$ is $O(g_2(x))$ then the function $f_1(x)f_2(x)$ is $O(max(g_1(x), g_2(x)))$.

T F If $a \neq 0$ and $ab \equiv ac(\mod m)$ then $b \equiv c(\mod m)$.

T F Given that $lcm(a, b) = 14$ and $a \times b = 42$ then necessarily $gcd(a, b) = 3$.

T F Consider the following two linear congruential generators, both of the form $x_{n+1} = ax_n + b(mod m)$.
(I) $a = 7$, $b = 4$, $x_0 = 3$ and $m = 9$
(II) $a = 7$, $b = 3$, $x_0 = 3$ and $m = 9$
The first generator is better.

T F The integer 111222333444 is divisible by 3 and 6 but not divisible by 9.

T F The complexity of the linear search algorithm is $O(n)$.

(2) (3 pts.) Show by using mathematical induction that for every nonnegative integer n, the integer $n^5 - n$ is divisible by 5.

(3) (3 pts.)Write the bubble sort algorithm and show the steps needed to sort the sequence $5, 7, 2$.

(4) (3 pts.) Given is the following procedure written in pseudo-code:

```
procedure lst(input: a1,...,an: integers)
L:=1
M:=a1
for i=2 to n
  if (ai > m) then
    begin
      m:=ai
      L:=i
    end
  else
  if (ai = m) then
    L:=i
{output: M and L}
```

Provide the output of this function when the input sequence $4, 4, 2, 3, 2, 4, 3, 1$ is entered, and state what the code does in general.

(5) (3 pts.) Use the Euclidean Algorithm to find $gcd(50, 14)$ and then represent it in the form $as + bt$.

(6) Given is the following procedure written in pseudo-code:

```
procedure locc(input: a1,...,an: integers)
m:= a1
for i=2 to n
   if (ai < m) then m:=ai
l:=0
for i=2 to n
   if (ai = m) then l:=i
{output: l}
```

(a) (2 pts.)Provide the output of this function when the input sequence $4, 4, 2, 3, 2, 4$ is entered, and state what the code does in general.

(b) (2 pts.)Calculate the number of comparisons if the above code as a function of the the input size n.

(7) (3 pts.) Show that a composite integer n must have a prime divisor less than or equal to $\sqrt{n}$.

(8) (3 pts.) Solve the congruence equation $15x \equiv 7(\mod 11)$

(9) (4 pts.) Find a function $g(n)$ so that $4n^2 - 5$ is $\Theta(g(n))$. Make sure you include the necessary witnesses.